Effect of Hole Area and Incline Angle on Pipe Flow Leakage Rates

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Abstract

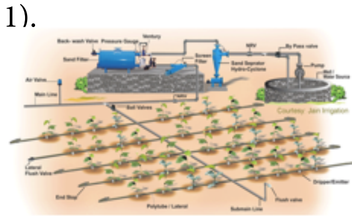
Water will always remain a valuable commodity due to its unique properties and availability. Therefore, its transport in pipes has great significance. Further, if leakage is controlled, an efficient mechanism of fluid administration can be created, as seen in common drip irrigation techniques. This study changed two variables of pipe perforation: hole area and pipe incline, and measured the resulting leakage rates. The experimental setup consisted of a pipe of varying hole areas attached to a water reservoir at varying angles. We hypothesized that for a horizontally configured pipe with a single hole, the leakage rate would increase linearly with hole area. The experimental data shows consistency with the hypothesis for a certain range of hole sizes but deviates from linearity outside this range. The study also presents a novel equation that models the correlation between pipe incline and leakage rate. The findings of this study provide more knowledge to incorporate variations to the drip-irrigation technique on both flat and oblique land.

Introduction

Industrial applications

Drip irrigation is an irrigation method that works by a leakage system. The fluid flows through a perforated pipe and is deposited at desired regions on a soil surface. In certain environments, the installation of an overhead sprinkler may not be practical due to limitation in resources. For example, in an agricultural field, the drip irrigation method is a more effective way of distributing both water and fertilizer. Compared to the overhead sprinkler, the system allows for lesser evaporation of solvent with field studies finding up to 90% efficiency, with reported reduction of diseases that arise by water contact [1].

Since monoculture fields grow similar species of plants with similar affinity for water next to each other, commercialized drip irrigation systems primarily adhere to a conventional horizontal design (Figure 1).



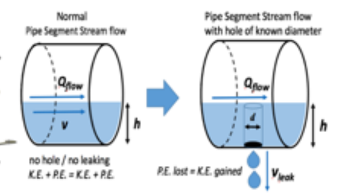


Fig. 1. (left) Typical drip irrigation system layout (figure reproduced without permission from Wikipedia [2].)
Fig. 2. (right) Side view of a pipe segment with constant flow rate Qflow, with (right) and without a hole (left).

This configuration allows for equal distribution of solvent at all points. However, fields may benefit from a non-horizontal pipe configuration, as inclined pipes deposit different amounts of leakage. Furthermore, if the land has a gradient, then knowledge about leakage rates in an angled pipe system can help distribute water to the desired amount.

Theory

Leakage from a hole of known diameter d

This experiment utilizes a horizontal pipe containing water traveling at a constant velocity (Figure 2). At the bottom of the pipe, there is a circular hole exposing the horizontal flow to a vertical channel causing leakage due to g, gravitational acceleration.

Torricelli's Law (Eq. 1), relates the velocity v_leak, of a fluid exiting a channel at the bottom of a reservoir filled to a depth h in the following form:

$$v_{leak} = \sqrt{2gh} \quad (1)$$

Since the variable at interest is the volume of leakage over time, Torricelli's law can be remodeled to give the classical discharge equation for circular orifice flow [3]:

$$Q_{leak} = k_d C\sqrt{2gh} \quad (2)$$

where Q_{leak} = leak rate (cm^3/s *or* mL/s)
k_d = the coefficient of discharge (*dimensionless*)
C = cross sectional area of the hole (cm^2)
g = acceleration due to gravity (cm/s^2)
h = height of the water level (cm)

The coefficient of discharge k_d accounts for energy lost from factors including boundary layer friction [4]. In the ideal condition, k_d is equal to 1.

We can solve the height of the water level h in the pipe once knowing the volumetric flow rate and the cross-sectional area of the water column. By measuring the time for a water body under a constant volumetric flow rate Q_{flow} to enter and exit a known length of the pipe segment L_{pipe}, we can compute the cross-sectional area A_{cross} of the water column:

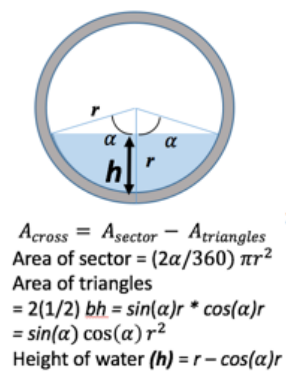
$$A_{cross} = Q_{flow}(t_{exit} - t_{enter})/\,L_{pipe} \quad (3)$$

As shown in Figure 3, the height h can then be solved analytically using geometry in two steps: First, substitute in the known radius r and A_{cross} into the equation $A_{cross} = A_{sector} - A_{triangles}$ to solve for the angle α. Next, solve the height of the water column via h=r-cos(α)r. In this experiment, the mean water height is calculated to be 0.7465 cm using this method. The height of the water column h stays relatively constant. Due to the conservation of volumetric flow rate, the leakage rate would theoretically change linearly to the area of the hole.

Inclination factor θ

The discharge equation is less accurate for predicting leakage rate when the stream is subjected to an incline. Therefore, the cross-sectional area A_cross and depth of water h must be reanalyzed.

Figure 4 shows a pipe with tube wall thickness ***S*** oriented at an angle, ***θ***, relative to a horizontal plane. Water flows at velocity ***v*** parallel to the orientation of the pipe. The original diameter of the hole, ***d***, is equivalent to the sum of ***d'*** and ***y***. Since ***y = S tan(θ), d' = d - S tan(θ).***



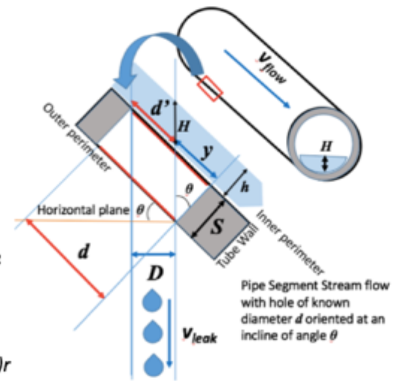


Fig. 3. (left) Cross Sectional View of Pipe and water height (h) estimation

Fig. 4. (right) Model of Inclined Pipe Leakage.

So, ***D*** is defined as the *effective diameter* in this configuration. The effective diameter is the actual diameter of the new leakage channel after the shrink in the horizontal hole size due to wall thickness and the incline is accounted for. Since $\cos(\theta) = \frac{D}{d'}$, **D** can be expressed as:

$$D = \cos\theta\,(d - S\tan\theta) \quad (4)$$

The effective area of the hole is no longer a perfect circle. Instead, it is an ellipse of area $\pi(\frac{d}{2})(\frac{D}{2})$, , where d/2 and D/2 are the semi major-axis and semi minor-axis respectively (Figure 5).

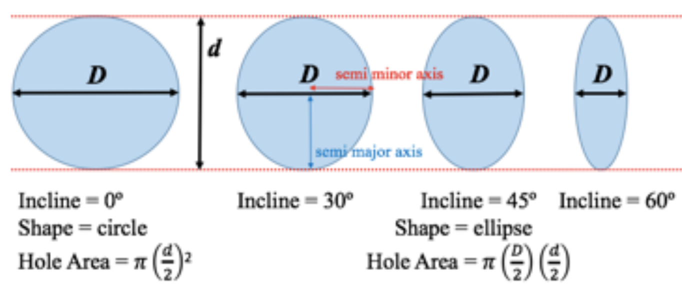


Fig. 5. The ellipse approach of effective hole area for inclined pipes.

All holes of the various inclined pipes all have the same major-axis, which is the original diameter of the hole. Their minor-axis is the effective diameter ***D***.

The height of water h in the pipe at any specific incline angle was estimated using the cross-sectional area given by Eq.3. Since t_{exit}- t_{enter} varies with angle, it becomes a variable and requires re-measurement. Similarly, the effective height of water column $\boldsymbol{H}$ replaces the perpendicular height by $\boldsymbol{H} = \frac{h}{cos(\theta)}$. Therefore, the proposed discharge equation that incorporates the structure of the inclined pipe is as follows:

$$Q_{leak} = k_e \pi \frac{d}{2} \left[\frac{cos\theta\,(d - S\,tan\theta)}{2}\right] \sqrt{\frac{2gh(\theta)}{cos\theta}} \qquad (5)$$

where Q_{leak} = leak rate (mL/s)
d = actual diameter of perforation (cm)
S = thickness of tube wall (cm)
$h(\theta)$ = angle-dependent height of stream flow in the pipe (cm)
θ = angle formed between the pipe and the horizontal plane (°)
k_e = experimental discharge coefficient

Methods

Materials

IPEX Inc. SDR21 10ft PVC tubes (inner, outer diameter = 23mm, 27mm)	Stopwatch
Dura Schedule 40 PVC ¾ inch adapter	5 m (± 1mm) Industrial Measuring tape
3ft Flexible Vinyl Tubing (diameter 5/8 inch)	30cm (± 0.5mm) Steel Ruler
IPS DeWALT 20V MAX Nail gun	100 mL (±1mL) Graduated Cylinder
15 RYOBI Speed Load Nails (diameters in inches): 1/16, 5/64, 3/32, 7/64, 1/8, 9/64, 5/32, 11/64, 13/64, 7/32, 15/64, 1/4, 9/32, 5/16, 3/8	The Home Depot ® Bucket
Glass Water Dispenser (2 Gallon / 7.6 L)	Step ladder
STANLEY Spirit Level	

Test facility

The experimental set-up (Figure 6) consists of a perforated pipe that is open on one end and connects to a water reservoir on the other end. The pipe connects to the reservoir via a vinyl tube, containers are placed under the hole and the open end to collect water.

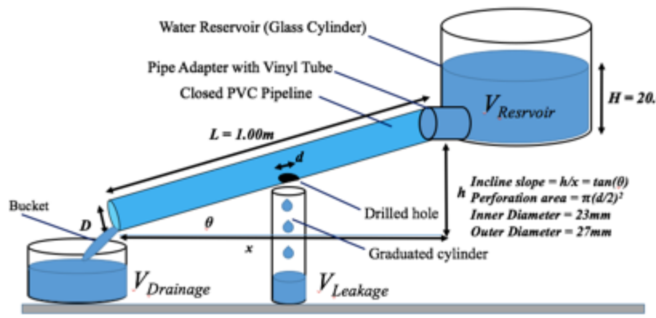


Fig. 6. Dimensional analysis of the experimental set-up. The two variables are the incline angle θ and hole diameter d.

Preparations and procedures

Pre-experimental preparations

Obtain 16 pipes of 1.00 m (± 0.01 m) length. Manually drill a single hole of known diameter at the 0.05 m mark from one end of each pipe segment. Account for any significant geometrical defects that occur inside the pipe such as residues that arise from drilling.

Place an empty reservoir on a leveled platform. Then, fill the reservoir with water until the 20.0 cm mark. Attach a faucet to the vinyl tube, the joint-adapter and the pipe in sequence. Make sure the hole on the pipe faces downward.

Experimental procedure

I. Altering hole area

Place a graduated cylinder beneath the hole to collect the leakage. Then, open the faucet and simultaneously start the timer. Close the faucet and simultaneously stop the timer when ~5 seconds have passed. Record the volume of water in the graduated cylinder. Refill the water in the reservoir back to 20.0cm high, and repeat this procedure five times to calculate the weighted mean leakage volume. Finally, repeat the procedure on pipes with different hole areas.

II. Altering pipe incline

Attach one end of the pipe to a pivot point on the table. Adjust the height of the other end of the pipe according to the desired angle, which is found using the trigonometric identity:

$$tan\theta = \frac{h}{x} \qquad (6)$$

Collect mean leakages for varying hole areas. Repeat procedure for each incline.

Uncertainty propagation

Since the measurement of leak rate depends on the volume of fluid collected over a time interval in the relationship $Q_{leak}=V_{leak}/t$, the divisional/fractional uncertainty principle [5] applies as follows:

$$\frac{\delta Q_{leak}}{Q_{leak}} = \sqrt{\left(\frac{\delta V_{leak}}{V_{leak}}\right)^2 + \left(\frac{\delta t}{t}\right)^2} \qquad (7)$$

The random error associated with recording time intervals on a stopwatch is assumed to be $\delta t = 0.11$s, as determined by taking the average reaction time of the experimenter. The uncertainty in volume was estimated by half of the last digit the graduated cylinder can provide at $\delta V = 0.5$mL.

Results and Discussion

Altering hole area

As Figure 7 suggests, there is a linear correlation between the leakage rate and the hole area, suggesting a direct proportionality ($Q \propto A$) in the discharge equation (Eq. 2). After trying to minimize the chi-square value of 4.91 obtained by a one-parameter fit, a two-parameter fit that included a y-intercept was tested. A resulting lower chi-square value of 0.46 was found.

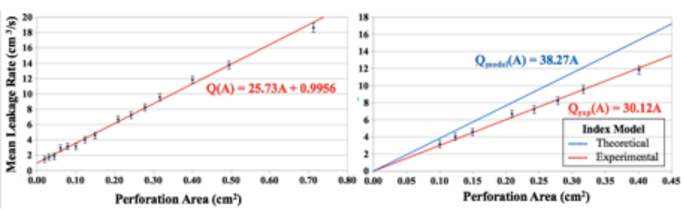


Fig. 7. (left) Mean leakage rate $\boldsymbol{Q}$ vs perforation area A for all 15 hole sizes, error bars represent relative uncertainties in $\boldsymbol{Q}$. (Red line: two-parameter fit model computed from least-square fitting with χ^2=0.46)
Fig. 8. (right) Leakage rate vs perforation area for 7 selected medium-range hole sizes, errors bars as $\boldsymbol{\delta Q}$. (Red: one-parameter fit model with χ^2=0.26; Blue: proposed linear fit from theoretical model with k_d = 1.00 and h = 0.7465cm)

However, the y-intercept that appeared in Figure 7 suggests that when the hole area is zero, leakage still occurs at a rate of 0.9956 mL/s, which is impossible. Theoretically speaking, as the hole area becomes infinitely small, there should be no water leaving due to effects such as adhesion and cohesion. Therefore, if hole diameters of a finitely small size (enough for where molecular interactions dominate) were tested, leakage rates would be expected to be zero. The linear property also deviates for oversized holes; as turbulent instability may be inevitable to the steady stream from the massive outflow of water through the hole underneath [6]. The larger the hole size, the greater the deviation would have resulted on the constant water height h inside the pipe.

Since non-extreme hole sizes are expected to follow the linear property, a second round of analyses (Figure 8) were done by including data from hole sizes between 0.1cm² and 0.4cm². χ^2 values for one-parameter and two-parameter models decreased to 0.26 and 0.11 respectively, which shows statistical significance in the model.

A further comparison between theoretical discharge slope with the experimental slope value suggests a non-ideal experimental discharge coefficient. k_e for this particular apparatus is found to be 30.12/38.27 = 0.7870 according to its definition:

$$k_e = \frac{actual\ discharge}{theoretical\ discharge} \qquad (8)$$

This value of the coefficient of discharge explains the difference found between our theoretical and our experimental linear models.

Altering pipe incline

To further investigate different properties of pipe leakage, pipe incline was introduced as a second variable. As the inclination angle increases, the relation between hole area and leakage rate becomes nonlinear.

The data taken for varying hole sizes and pipe inclines were superimposed into one graph in Figure 9, which shows the change in leakage rate at various tilting angles. There was generally a nonlinear decrease in the leakage rate as the incline slope increased. For all hole diameters tested, the drp in Q_{leak} was measured to be greatest for the smallest angle change from 0° to 4°. As the pipe becomes more tilted, the leakage rate continues to decrease but at a slower rate. The trend lines indicate that at higher incline slopes, Q_{leak} would continue its slow decreasing tendency until it approaches zero at some threshold incline when the effective hole area becomes zero. . For smaller holes, the leak-free condition occurs at a smaller incline. “8/64 inch” was the only hole diameter that has achieved this at an angle of 20°.

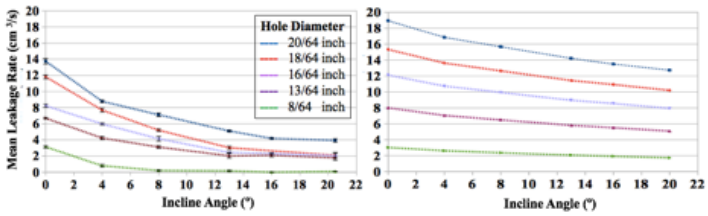


Fig. 9. (left) Mean leakage rate Q_{leak} vs Incline angle $\boldsymbol{\theta}$ for various hole-size pipes, measured at five tilting angles (4°, 8°, 13°, 16°, 20.5°), error bars represent the standard deviation from five independent measurements.
Fig. 10. (right) Predicted leak rate $\boldsymbol{Q}$ as a function of incline angle $\boldsymbol{\theta}$ for the same five hole sizes tested, based on the theoretical model of Eq. 5.

The theoretical model based on the proposed incline discharge equation (Figure 10) showed a similar decrease trend in Q_{leak} as a function of angle $\boldsymbol{\theta}$ with the greatest drop experienced in smallest angles. However, the rate of decrease in Q_{leak} is much slower from the model and suggests none of the tested hole sizes would reach leak-free standard until a very high incline angle relative to the test range. This discrepancy is likely due to the fact that the proposed model does not account for the attraction of liquid molecules to each other due to cohesion, and the attraction between liquid molecules and the inner wall of the pipe due to adhesion. This may be a large factor causing the sharp decrease in leakage rates. Therefore, the difference in the theoretical and experimental data can be explained due to the specificity of the experimental set-up; if larger holes were used, effects such as adhesion and cohesion would be minimized, and the experimental data would reflect the theoretical more accurately.

Uncertainty propagation

Error analysis in constant flow-rate assumption

The discharge equation used in the study only results in a constant leakage rate if the volumetric flow rate is constant. Since the experiment utilized a water reservoir as the source of flow, this is not the case. There is a small degree of variation that arises from the dependence of water height in Torricelli's equation. As the water leaves the reservoir and enters the pipe, the water height decreases, and the flow rate continuously decreases.

This difference was found to be negligible after an uncertainty propagation. On average, in the 5s interval that was used to collect the leakage, 0.268 L, or 2.68 x 10-4 m^3 of water left the reservoir. The reservoir had an inner circumference of 0.660 m, therefore a cross-sectional area of 0.0347 m^2. Thus, the difference in height due to the leakage would approximately be 7.22×10^{-3} m. By calculating the flow rates at the initial height (0.200 – 0.055 = 0.145m), and final height accounting for the decrease in height (0.145 – 7.22 x 10-3), it was found that the flow velocity only experienced a slight decrease from 1.687 m/s to 1.642 m/s as water entered the pipe over the 5s interval. This accounts for a maximum omission of 0.299mL/s of volumetric flow rate, below the 0.41mL/s minimum uncertainty of δQ.

Error analysis in perforation area

The information for the hole area in each measurement is not a result of direct measurement of the physical perforation but rather determined from the diameter labeled on each drill bit used to drill these holes prior to the experiment, which introduces uncertainty. Therefore, improving the drilling technique may ensure a more accurate hole area estimation.

Conclusion

The study has successfully found theoretical models that fit experimental data, which provide a better understanding of leakage rates when subjected to varying hole areas and pipe inclines. For an open channel pipe segment with a constant volumetric flow rate, the leakage rate for a single circular hole located at the bottom of the pipe wall was verified to be directly proportional to the area of the hole. The experimental data for leakage rates resulting from varying incline angles partially reflected the proposed incline discharge equation, but not fully, due to factors such as intermolecular interactions, which were not accounted for. Therefore, future studies should investigate the effect of increasing pipe incline angle on leakage rates with greater hole sizes (while maintaining a low hole diameter to pipe diameter ratio), where macroscopic effects dominate over intermolecular interactions.

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Drill Bit Diameter d [inch]	Hole Radius r [cm]	Hole Area A [cm^2]	Experimental Leak Rate Q_{exp} [mL/s]	Relative Uncertainty δQ_{exp} [+/- mL/s]	Standard Deviation σ [+/- mL/s]	Model Leakage Rate Q_{model} [mL/s]
4/64	0.08	0.02	1.49	0.41	0.04	0.76
5/64	0.10	0.03	1.75	0.41	0.06	1.18
6/64	0.12	0.04	1.87	0.41	0.04	1.70
7/64	0.14	0.06	2.92	0.41	0.05	2.32
8/64	0.16	0.08	3.14	0.41	0.07	3.03
9/64	0.18	0.10	3.15	0.41	0.04	3.83
10/64	0.20	0.12	4.02	0.42	0.10	4.73
11/64	0.22	0.15	4.58	0.42	0.10	5.73
13/64	0.26	0.21	6.69	0.43	0.04	8.00
14/64	0.28	0.24	7.22	0.44	0.15	9.28
15/64	0.30	0.28	8.24	0.45	0.20	10.65
16/64	0.32	0.32	9.56	0.46	0.15	12.12
18/64	0.36	0.40	11.82	0.48	0.12	15.34
20/64	0.40	0.49	13.76	0.51	0.17	18.94
24/64	0.48	0.71	18.63	0.58	0.21	27.27

Table. 1. Experimental mean leakage rate Q_{exp} and theoretical leakage rate Q_{model} suggested by Eq. 2 for various hole sizes. Standard deviation, σ, computed over five measurements for each hole size. $\boldsymbol{\delta Q}$ established by Eq. 7