ARTICLE
Accuracy of the Binomial Asset Pricing Model Using Daily Volatilities

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INTRODUCTION

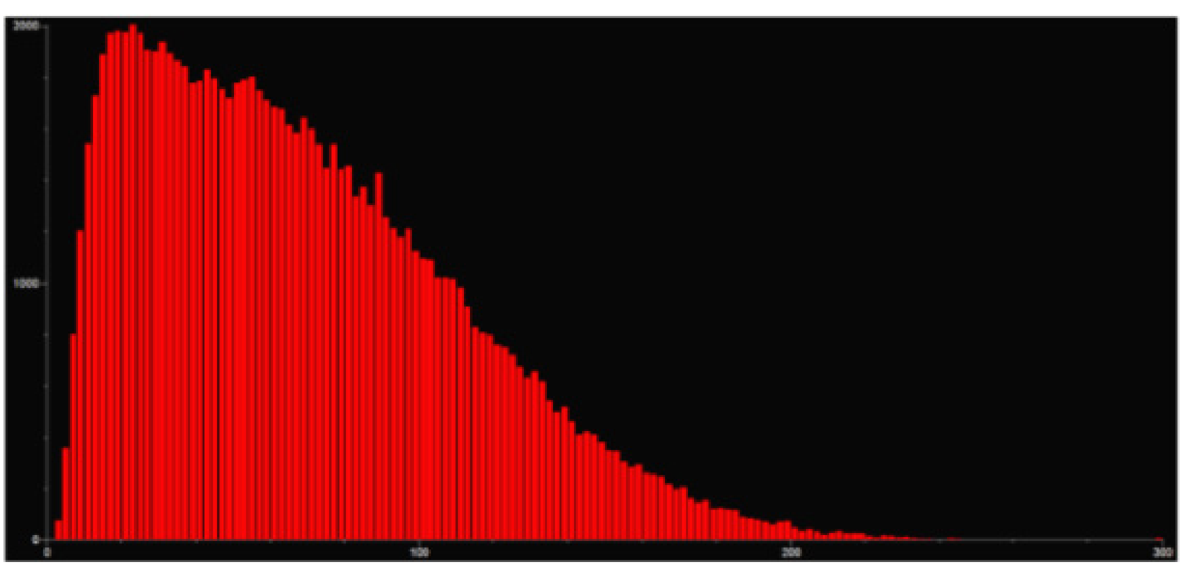
The binomial asset pricing model is well established, having been developed in 1979 by John C. Cox, Stephen A. Ross, and Mark Rubinstein [1]. In their paper "Option pricing: a simplified approach", they propose that stock prices can be modeled by probabilities of price increase or decrease by a set amount in a time period depending on the volatility of the stock [1]. While they used constant volatilities for their calculations, it has been identified by many other scholars that the assumption of constant volatility reflects the market poorly [2, 3], and that stochastic volatility serves the purpose better. This article compares the accuracy of the model stock prices using daily volatility values with using a constant volatility to historical stock prices in attempts to show that daily volatility rates represent the market better.

DISCUSSION

The binomial asset pricing model assumes that stock prices "S" will either increase or decrease a fixed amount each set trading period [4]. The probability of increase "p" (1), the stock price increase "u*S" (2), and the stock price decrease "d*S" (3) are all dependent on the stock volatility "σ". The time increment "δt" represents the amount of time between each price change in the model. The risk-free continuous compounding rate of return "r f " represents the average rate of return from a very stable investment. This article had used the average 10 year Canadian Treasury bill return rate for r f , which is 6.4% [5].

A random number generator was used to determine whether the stock price would go up or down, depending on whether the number was bigger or smaller than p respectively, to simulate the stochastic process of price changes. Daily volatilities were calculated by taking the standard deviation of the return rate "r" (4) of the stock over a period of time (5).The daily volatilities used were calculated using the return rates of the previous 5 trading days in Eq.

Figure 1. Histogram of the X^2 values generated from 100,000 runs of the binomial asset model for Apple with daily volatilities. The x-axis represents the X^2 values and the y-axis represents the frequency of occurrence of X^2 values that fall within discrete intervals of size 2.



5. Stocks were modeled given their initial open prices “S0”, their volatilities, the time period between price changes “δt”, and the total amount of time for the model to run.

To show that daily volatilities would reflect stockprices better than a constant average volatility in the binomial asset pricing model, X 2 quantify the accuracy of the model, where “Shis” was the historical stock price and “Smol” was the model stock price. X 2 used because there were discrete numbers of data points generated. The X 2 recorded was the average of the X 2 from one run of the model, so it reflects the accuracy of a single execution of the model.

The confidence was determined by the 3 sigma uncertainty “σX” (7), of the X 2 where this value represents the consistency of the accuracy of the model. The σX was calculated from the array of X2 by running the model “a” times.

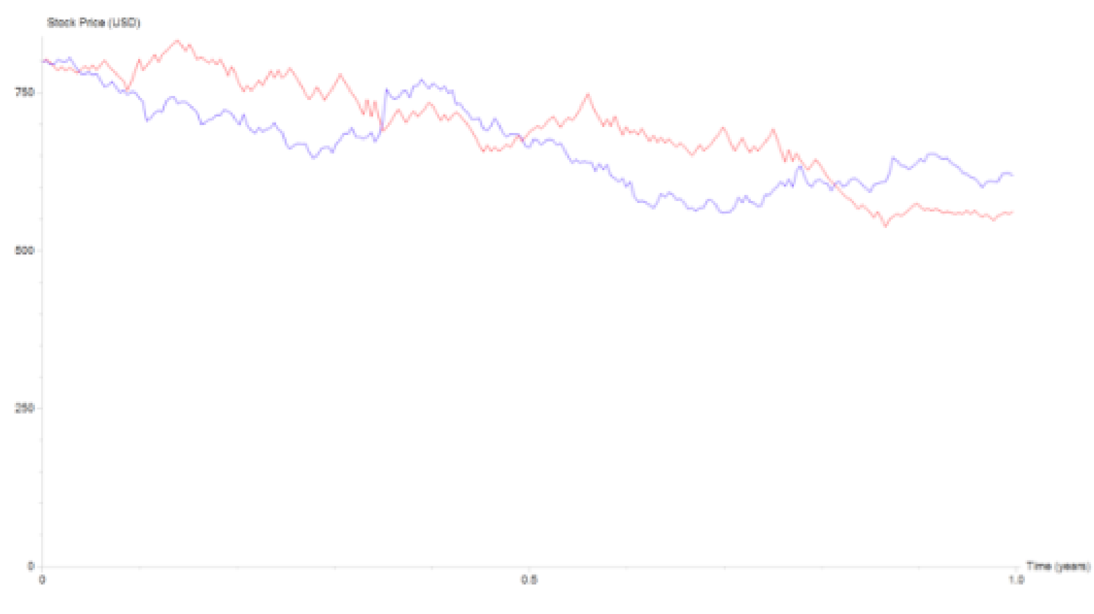
To distinguish the distribution of X 2 values, the Anderson-Darling test was used to test for normality [6]. The results of the test showed that the distribution of X 2 values does not follow Gaussian or lognormal distribution. The distribution of values can be seen in Figure 1.

From Figure 1, the distribution appears to be skewed. The shape of the distribution looks like half of a normal distribution, although there was a drop of the frequency of X 2 can be explained by the fact that X 2 were always positive and that they were non-zero because it was very unlikely that the model matches the historical data exactly. The confidence “C” of the uncertainty measurement can be calculated by integrating the distribution curve, by modeling it with a function “f(x)” (8), as seen in Eq. 9. “f(x)” has scalable quantities “ω” and “τ” in order for it to better fit the distribution. Although f(x) does not match the distribution of X 2 error was negligible as it was too small.

Using this method of analysis, the model was run one million times with daily volatilities and with a constant average volatility, over the period of time which the stock was to be modeled, to determine the average X 2 stocks, Apple, Google, General Motors, CIBC, and RBC. The historical volatility and stock prices used were from March 2012 to March 2013 for all these stocks.

There appears to be a general trend that the model data fits the historical data better within a shorter period, from where it diverges away. Reading the graphs of the model data with the historical stock price does not provide a clear indication of whether using daily or constant average volatility values were better. An example of the graphs produced can be

Figure 2. Graph of the predicted stock price, in red, using non-constant volatilities of Google stock with the actual stock price in blue for reference. The data was taken from March 2012 to March 2013.



seen in Figure 2. The results of the model runs were summarized in Table 1. “C” was calculated for all the stocks, and the average “C” was reported in Table 1 as well.

It seems that in Table 1, the uncertainty for X 2 constant volatility were always smaller than the uncertainty for X 2 using daily volatility values. This may be the result of the fluctuations in the probability of price increase when non-constant volatility values were used, because there were more permutations of stock values compared to when there is a constant probability of price increase.

It also seems that there was a difference in X 2 as Google and Apple in Table 1. This was because the X 2 price changes, and higher stock prices result in bigger price changes because price changes were proportional to the initial stock price. Hence X 2 greater for stocks with higher stock prices.

In conclusion, it also appears that using daily rather than constant volatility rates, as seen in Table 1, decreases the accuracy of the binomial asset pricing model significantly with 99.6% confidence, because the X 2 the σX found. One way to explain why using daily volatility rates may result in a decrease in accuracy compared to using a constant average volatility rate was because by keeping p updated with current volatility rates, the price should reflect similar degrees of changes with historical prices.

However in periods of high volatility, the u and d values are also very high or low so the model price can move away from the historical price. Then if a period of low volatility follows, the u and d values approach closer to 1 again, so it would be difficult for the model price to converge to the historical price, and it would remain far away from the historical prices. Having a constant volatility means that there would be constant u and d factors. Hence the model with a constant volatility would have a better chance of converging towards the historical price, resulting in lower X 2 .

The model was imperfect in both cases, when using an average volatility value and when using daily volatility values. One of the major reasons why the model prices do not match historical prices was because much of stock prices are governed by supply and demand, where the majority of investors may be relying on other sources, such as a Q4 report, as an indication of whether the stock will provide satisfactory returns. These other sources may contradict what the model was proposing, so the model will move away from the market stock prices.

SUMMARY

The binomial asset pricing model was more accurate when an average volatility was used rather than daily volatilities. Periods of high volatility may explain this phenomenon. As big changes occur in the stock price during periods of high volatility, the volatility drops low again so any previous big changes made will be hard to reverse, resulting in a significant gap between the model stock price and the historical stock price. This result suggests that using daily volatility values in the binomial asset pricing model may not portray a stock value as accurately as an average volatility would in the binomial asset pricing model with 99.6% confidence. It is important to realize this fact, although it may seem counterintuitive, because derivative or stock traders can continue

Table 1. Summary of the X^2 values and the 3 sigma uncertainties of the X^2 values which represent the accuracy of the binomial asset pricing model for when constant volatility and non-constant volatility rates were used. Average confidence “C” was 99.6%.

Ticker Name	Company	Constant Volatility		Daily Volatility	
		X^2	σ_X	X^2	σ_X
NASDAQ: GOOG	Google	18.90192278	0.054569335	22.71550222	0.069359596
NASDAQ: AAPL	Apple	66.08307438	0.118808071	68.79427304	0.128888542
NYSE: CM	CIBC	0.930710811	0.002492370	1.18358985	0.003446640
NYSE: RY	RBC	1.068702943	0.002752178	1.278136009	0.003478875
NYSE: GM	General Motors	1.40828497	0.003534972	1.575716295	0.004277933

using average volatility values in their pricing models in order to increase the accuracy of their price predictions. Suggestions for future research topics include why using daily volatility values results in less accuracy and precision in predicting stock prices, or how bigger volatility fluctuations affect the binomial asset pricing model.

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